

Remarks on the proton charge radius

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Abstract

Proton charge radius is calculated from the electromagnetic form factor of proton parameterized by the dispersion relation. The calculated charge radius is a little larger than that obtained by the Lamb shift of the μ mesic atom. As the result is sensitive to the experimental data of proton electric form factor at small momentum transfer, more accurate data are required to draw conclusion if the result of the nucleon form factor is different or not from that obtained Pohl et al.

Recently, very accurate proton charge radius has been reported by Pohl et al. obtained by the measurement of Lamb shift of the μ mesonic atom [1]. The proton charge radius is smaller than that obtained by the electron proton scattering; they report the proton charge $r_p = 0.84184(67)$ fm, while $r_p = 0.96276$ fm when the proton electric form factor is approximated by the well-known dipole formula, being very good approximation for the small momentum transfer. Flowers emphasized the possibility that the discrepancy may lead to the break down of QED [2].

The mean square charge radius obtained by the electron proton scattering is given in term of the Fourier transform of the form factor by a simple formula

$$\langle r^2 \rangle = \lim_{Q^2 \rightarrow 0} \left[-6 \frac{d}{dQ^2} F(Q^2) \right]. \quad (1)$$

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To obtain the experimental value for the proton charge radius numerical differentiation should be performed.

The experimental data of the electromagnetic form factor are given for small Q^2 by Borkowski et al. [3] and Simon et al. [4], which are illustrated Fig.1. To obtain the experimental value for the charge radius it necessary to calculate Eq.(1) for the experimental data. Sick [6] and De Rúja [7] approximated the experimental data [4] by analytic functions and performed numerical differentiation. They obtained the following proton charge radius: $\sqrt{\langle r^2 \rangle} = 0.895 \pm 0.018$ fm. The result is a little larger than that of Pohl et al. Considering the error, De Rúja conclude the that QED is not endangered by the experiment of Pohl et al. It must be noted that the numerical differentiation is dependent on how the experimental data are approximated.

We have performed analysis of nucleon electromagnetic form factors by using the dispersion relation

$$F_1(Q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im}F_1(s)}{s + Q^2}, \quad (2)$$

where F_1 is the Dirac form factor, m_π is the pion mass, s is the squared energy for the nucleon and antinucleon system and Q^2 is the squared space-like momentum transfer. F_1 , given by Eq.(2), satisfies appropriate properties such as low energy hadronic properties, the vector boson dominance and QCD constraints [8]. We were able to parameterize the exiting electromagnetic form factors of nucleons, for the space-like and the time-like momenta as well and very good agreement with the experimental data was obtained. It is the purpose of this paper to calculate the proton charge radius by our dispersion relation. In our analysis we used the data of Borkowsski el al. [3] but omitted Simon el al. [4] as the errors of the latter are random errors only. We thought the data were not appropriate in performing the chi square test.

Before discussing the numerical results, we give some remarks on the form factor. As the well-known dipole formula $G_D = 1/(1 + Q^2/0.71)^2$, with Q^2 expressed in terms of GeV^2 represents the experimental data of G_E^p fairly well, the proton charge radius corresponding to the dipole formula is taken as a standard. The experimental data and the theoretical results as well are given as the ratio to G_D . When G_E^p/G_D is substituted to Eq.(1), we get the difference $\langle r^2 \rangle|_{G_E^p} - r_D^2$, where r_D is the charge radius corresponding to the dipole formula. If the experimental data G_E^p/G_D becomes larger than 1 near $Q^2 \approx 0$, $\langle r^2 \rangle|_{G_E^p}$ becomes smaller than r_D^2 .

We have considered the Sachs electric form factor so far, but we have the Dirac charge form factor F_1^p . It must be noted that the proton charge radius is to be calculated by using F_1^p in (1). Writing the Charge form factor in terms of the Sachs form factors, we have

$$F_1^p(Q^2) = \left(G_E^p + \frac{Q^2}{4m^2} G_M^p \right) / \left(1 + \frac{Q^2}{4m^2} \right), \quad (3)$$

where m is the proton mass. Therefore,

$$\langle r^2 \rangle|_{F_1^p} = -6 \frac{d}{dQ^2} F_1^p(Q^2)|_{Q^2=0} = -6 \frac{d}{dQ^2} G_E^p|_{Q^2=0} - \frac{3g^p}{2m^2}, \quad (4)$$

where g^p is the proton anomalous magnetic moment, $g^p = 1.79284939$. Thus we have

$$\langle r^2 \rangle|_{F_1^p} = \langle r^2 \rangle|_{G_E^p} - \frac{3g^p}{2m^2} \quad (5)$$

in GeV unit.

Let us give our calculations obtained by the dispersion relation [8], where the proton data of Simon et al. [4] were not used as was mentioned before. In this paper we take account of Simon et al., and leave out Borkowski et al., as the former is more accurate for small Q^2 . We investigate how the proton charge radius change by this modification.

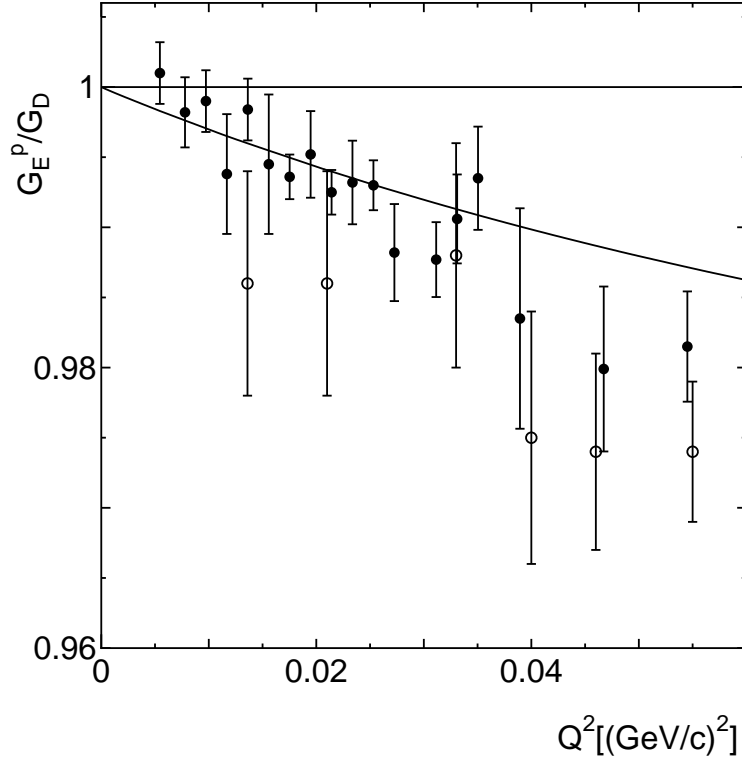


Figure 1: The theoretical curve obtained by replacing Borkowski et al. [3] by Simon et al. [4]. Closed circles represent Simon et al. [4] and the open circles Borkowski et al. [3].

We give our numerical values of the proton charge radius calculated by our dispersion relation. It must be noted that the data points are the same

as in Ref. [8] except for the proton charge form factor at low Q^2 . The case II of [8] is used.

First we give the proton charge radius calculated by using the parameters determined in Ref. [8].

$$\langle r^2 \rangle|_{G_E^p} = 25.751 \text{ GeV}^{-1}.$$

When the data of Borkowski et al. [3] are omitted and Simon et al. [4] are added we have the following result:

$$\langle r^2 \rangle|_{G_E^p} = 24.5551 \text{ GeV}^{-1}.$$

Our calculated result agree with data of Simon et al. rather than Borkowski et al. The total value of χ^2 , addition of space-like and the time-like part, becomes $\chi_{tot}^2 = 505.6$ after the replacement of Borkowski et al. by the Simon et al. Our previous value of Ref.[8] is 524.5. We illustrate in Fig.1 the calculated result together with experimental data.

The proton charge radius is thus obtained to be

$$\sqrt{\langle r^2 \rangle|_{F_1^p}} = 0.9401 \text{ fm}, \quad (6)$$

with the experimental data and the parameters given in [8], and

$$\sqrt{\langle r^2 \rangle|_{F_1^p}} = 0.9149 \text{ fm}, \quad (7)$$

where the data of Borkowski et al., are replaced by Simon et al.

The charge radius of proton is a little larger than the result of Pohl et al. However it is premature to conclude that the QED is endangered by their experiment, as the proton charge radius obtained from the electromagnetic form factor is sensitive to the low Q^2 part of the form factor. More accurate data near $Q^2 \approx 0$ are required.

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